Supplement to "Locally Ideal Formulations for Piecewise Linear Functions with Indicator Variables"

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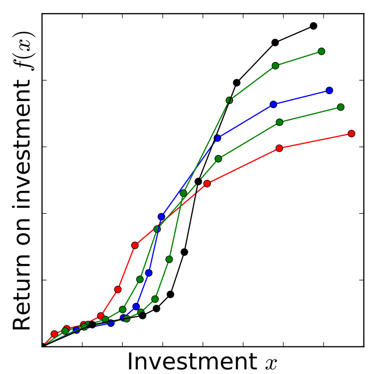
Data generation

The three components of the instances used are the return on investment functions $f_{jk}(\cdot)\ \forall j \in \mathcal{J}\ k \in \mathcal{K}$, fixed costs $G_j\ \forall j \in \mathcal{J}$ for entering each product market and variable costs $c_{jk}\ \forall j \in \mathcal{J},\ k \in \mathcal{K}$ per unit of budget allocated for marketing strategy $k \in \mathcal{K}$ of product $j \in \mathcal{J}$. We now describe how each of these components were generated.

Return on investment functions

In our sample application, the return on investment is evaluated by piecewise-linear functions $f_{jk}(\cdot)$ which have the typical form shown in Figure 1.

Figure 1: Sample curves modeling return on investment for five different product/strategy pairs.



Let $R(a, b, i, n)$ denote a random variable that lies between $a + \frac{i(b-a)}{n}$ and $a + \frac{(i+1)(b-a)}{n}$ with a distribution $a + \frac{b-a}{n}(i + \beta(2, 2))$ where $\beta(2, 2)$ is the beta distribution with both parameters set to 2. For each product $j \in \mathcal{J}$, the domain of $f_{jk}(\cdot)\ \forall k \in \mathcal{K}$ was generated using

$$d_j \sim R(4, 8, j, |\mathcal{J}|)$$

and the range was generated as

$$r_j \sim R(0.5, 1, j, |\mathcal{J}|)$$

where the notation j overloads both the product $j \in \mathcal{J}$ and an unique index for the product between 1 and $\mathcal{J}$. The desired s-shaped functions were generated by dividing the domain $[0, d_j]$ of $f_{jk}(\cdot)$ $\forall k \in \mathcal{K}$ into three parts such that $f_{jk}(\cdot)$ is concave increasing in $[0, a^1_{jk}]$, convex increasing in $[a^1_{jk}, a^2_{jk}]$ and concave increasing again in $[a^2_{jk}, d_j]$. The random variables a^2_{jk} and $a^2_{j,k}$ were generated using

$$\begin{aligned} a^1_{jk} &\sim d_j R(0.1, 0.5, j, |\mathcal{J}|) \\ a^2_{jk} &\sim d_j R(0.3, 0.7, j, |\mathcal{J}|). \end{aligned}$$

The set of breakpoints B_{jki} $\forall i \in \{1 \dots n\}$ were calculating by dividing each of the three domains into approximately $\frac{n}{3}$ equal parts which can be written as

$$\begin{aligned} B_{jki} &= 3i\frac{a^1_{jk}}{n} && i = 1 \dots \left\lfloor \frac{n}{3} \right\rfloor \\ B_{jki} &= a^1_{jk} + 3i\frac{a^2_{jk} - a^1_{jk}}{2n} && i = \left\lfloor \frac{n}{3} \right\rfloor + 1 \dots \left\lfloor \frac{2n}{3} \right\rfloor \\ B_{jki} &= a^2_{jk} + 3i\frac{d_j - a^3_{jk}}{n} && i = \left\lfloor \frac{2n}{3} \right\rfloor + 1 \dots n. \end{aligned}$$

The corresponding function evaluations $F_{jki} := f_{jk}(B_{jki})$ were generated as

$$\begin{aligned} F_{jki} &= b^1_{jk}\sqrt{\frac{B_{jki}}{B_{jk\lfloor \frac{n}{3} \rfloor}}} && i = 1 \dots \left\lfloor \frac{n}{3} \right\rfloor \\ F_{jki} &= F_{jk\lfloor \frac{n}{3} \rfloor} + b^2_{jk}\left(\frac{B_{jki} - B_{jk\lfloor \frac{n}{3} \rfloor}}{B_{jk\lfloor \frac{2n}{3} \rfloor} - B_{jk\lfloor \frac{n}{3} \rfloor}}\right)^2 && i = \left\lfloor \frac{n}{3} \right\rfloor + 1 \dots \left\lfloor \frac{2n}{3} \right\rfloor \\ F_{jki} &= F_{jk\lfloor \frac{2n}{3} \rfloor} + b^3_{jk}\sqrt{\frac{B_{jki} - B_{jk\lfloor \frac{2n}{3} \rfloor}}{B_{jk\lfloor \frac{2n}{3} \rfloor} - B_{jk\lfloor \frac{n}{3} \rfloor}}} && i = \left\lfloor \frac{2n}{3} \right\rfloor + 1 \dots n \end{aligned}$$

where b^1_{jk}, b^2_{jk} and b^3_{jk} are random variables distributed by

$$\begin{aligned} b^1_{jk} &\sim r_j R(0.05, 0.1, j, |\mathcal{J}|) \\ b^2_{jk} &\sim r_j R(0.4, 0.7, j, |\mathcal{J}|) \\ b^3_{jk} &\sim r_j R(0.7, 1, j, |\mathcal{J}|). \end{aligned}$$

Costs and Budget

For each strategy $k \in \mathcal{K}$ and product $j \in \mathcal{J}$, the per-unit operating costs were generated as

$$c_{jk} \sim \beta(2, 2)\ R(0.8, 1.2, j, |\mathcal{J}|)\ R(0.8, 1.2, k, |\mathcal{K}|) \qquad \forall j \in \mathcal{J}, k \in \mathcal{K}$$

and the fixed costs were generated as

$$G_j \sim E_G\ R(0.5, 1, j, |\mathcal{J}|)\ \mathrm{U}(0.8, 1.2) \qquad \forall j \in \mathcal{J}.$$

where $E_G = 0.105|\mathcal{J}|\ |\mathcal{K}|$. This procedure ensured that the total fixed costs are of the same order as the total variable costs. The overall budget D was set to $6E_G$.